Dynamic Fractional Frequency Reuse in OFDMA systems

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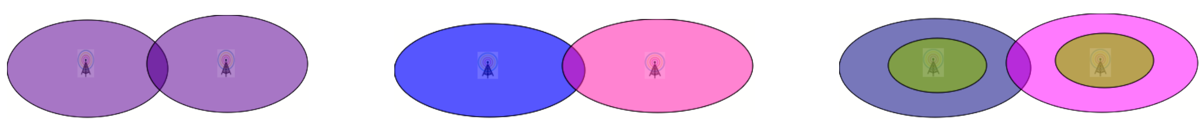
1 Introduction

While OFDMA orthogonalizes users within a cell, out-of-cell interference avoidance is a primary concern in the practical implementation of OFDMA systems. This project explores one of the key ideas towards limiting the harmful effects of out-of-cell interference: fractional frequency reuse (FFR) (1). We first explore the idea of frequency reuse and introduce FFR. Then, we discuss the algorithm for dynamic FFR proposed in (2). Finally, we analyze the performance of the scheme though simulations, comparison with a toy model, and the resulting outage probability.

2 Frequency Reuse



(a) Reuse-1 (b) Reuse-$\frac{1}{2}$ (c) Fractional frequency reuse

Figure 1: Frequency Reuse Schemes

Let us examine a setup with only two neighboring cells. An easy scheme that avoids inter-cell interference is the reuse-$\frac{1}{2}$ scheme (Figure 1b), which divides the total bandwidth between the two cells. In this case, users see no inter-cell interference at the cost of reduced bandwidth. On the other hand, reuse-1 (or universal reuse, Figure 1a) is a proposition where users in each cell have access to the entire bandwidth but must deal with interference from out-of-cell users. A toy example provides further insight into the tradeoff between bandwidth, power and

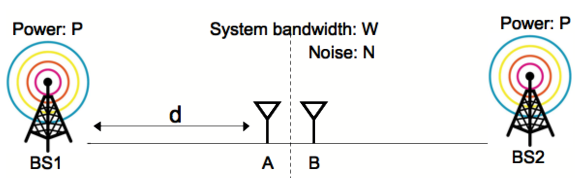


Figure 2: Two-cell, two-subband, two-user example

interference (Figure 2). Consider two cells with base stations (BSs) distance l apart, each constrained by a total power constraint P and system bandwidth $2W$. Alice and Bob are in cell 1 and cell 2 at distance d from their respective BS. We ignore fading for the sake of simplicity. The system capacities achieved per user by the reuse-1 and reuse-$\frac{1}{2}$ schemes are (with path loss model $pl(d) = 133.6 + 35 \cdot \log(d)$):

$$\text{Reuse-1} \qquad 2W \cdot \log(1 + \frac{\frac{P}{2} \cdot pl(d)}{WN + \frac{P}{2} \cdot pl(l-d)})$$

$$\text{Reuse-}\frac{1}{2} \qquad W \cdot \log(1 + \frac{P \cdot pl(d)}{WN})$$

These capacities per user vary with d as shown in Figure 3. The green and blue surfaces show the capacity achieved under reuse-1 and reuse-$\frac{1}{2}$ respectively. As the distance of each user from its own BS decreases and the total power available to each BS increases, reuse-1 starts outperforming reuse-$\frac{1}{2}$. This shows that adopting any one fixed strategy in ignorance of user geography would be suboptimal in many regimes.

The fractional frequency reuse (FFR) scheme accounts for user geography during resource allocation. Figure 1c shows an example of FFR in a two-cell scenario. While cell interior users who face little out-of-cell interference enjoy reuse-1, cell edge users mitigate out-of-cell interference by employing a reuse-$\frac{1}{2}$ strategy. However, even this strategy involves the static classification of users into one of two categories and cannot deal with changing user traffic or user locations. Dynamic FFR (DFFR) offers the flexibility necessary to adapt to a changing environment.

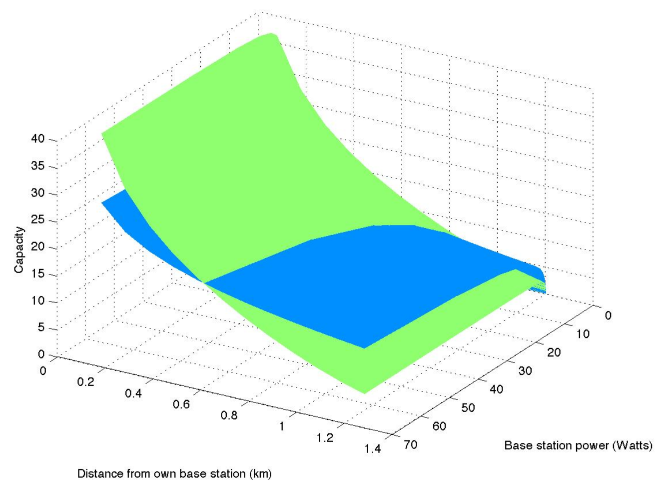


Figure 3: Comparison of reuse-1 and reuse-$\frac{1}{2}$ capacities in the two cell toy example.

3 Dynamic Fractional Frequency Reuse

Stolyar and Viswanathan (2) propose an implementation for DFFR, which includes two main ideas. The first is a heuristic, distributed algorithm for optimal resource allocation in a multi-cell setting. With no communication between base stations each BS dynamically allocates resources to each of its users. Optimizing resource allocation to maximize capacity is known to be an NP-hard problem (3). However, instead of dealing with this, in (2) a BS allocates resources to users so as to minimize the total power consumed while maintaining a constant data rate (CDR) to each user. The second idea introduced in the paper is that of grouping subcarriers into *subbands*. A subband is simply a set of subcarriers. For example, consider grouping 48 subcarriers into three sets of 16. For a user to be placed in a particular subband means that his virtual channel may include only frequencies that are part of that subband. We explain below how the use of subbands can improve system performance.

In (2), the following assumptions are made. An OFDMA system is used with inter-cell interference but no inter-carrier interference. Users desire no more than the constant data rate R. Each user periodically and truthfully feeds back channel quality information for each subband.

From the channel quality information that the base station receives for each user i, it calculates the number of subcarriers (m_{ij}) and the amount of power (p_{ij}) the user would need *if user i were placed in (and achieved rate CDR on) subband j*. Let z_{ij} be a binary-$(0,1)$ random variable which denotes whether user i is assigned to subband j. Then, each base station needs to solve the following constrained integer programming problem, where c is the subband capacity and p is the total power constraint:

$$\min_{z_{ij}} \sum_{i,j} p_{ij} z_{ij} \tag{1}$$
$$\sum_{i,j} p_{ij} z_{ij} \leq p$$
$$\sum_{i} m_{ij} z_{ij} \leq c \quad \forall j$$
$$z_{ij} = 0 \text{ or } 1$$

They further relax this by allowing the z_{ij} to be real in the interval [0,1] and $\sum_j z_{ij} = 1,\ \forall i, \quad z_{ij} \geq 0 \quad \forall i,j$ In practice the z_{ij}'s tend to be 0 or 1. The details of the allocation can be found in (2). Here, we provide only the intuition behind the algorithm. Consider again a two-cell scenario as in Figure 2. If cell-edge users in cell 1 are primarily allocated to subband 1, users in cell 2 will see high interference on subband 1 and will thus naturally be pushed towards subband 2. Cell interior users are not as affected by inter-cell interference, and hence will end up choosing the subband with the best fade.

Users are considered in a sequential order. At this point, the base station evaluates the user's cost[1] (a function of power) on each subband and chooses the cost-minimizing subband. The base station's state variables (which monitor available subbands and power) are subsequently updated to reflect this assignment and then the next user is considered. At the end of each allocation period the state variables are decremented to represent work orders being processed.

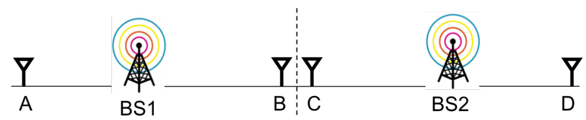


Figure 4: Two-cell, two-subband, four-user example

This scheme thus allows a user to choose the "best" subband for itself. At a high level this might seem very similar to universal reuse since there is no strict allocation of frequency bands among cells. It is not immediately clear why this scheme should yield increased performance over universal reuse (as (2) claims). However, the effect of the subbands is to organize users into groups which interfere the least with one another. Consider the example in Figure 4 with two subbands. For simplicity, assume that there is path loss but no fading. OFDMA with universal reuse would have users A and B each interfering with user C 50% of the time. However, user C would much prefer to see the interference from user A 100% of the time (and never experience any interference from user B). This will happen in the subband situation since the virtual channels for B and C will never coincide.

4 Implementation and Results

We contacted the authors of (2) in an attempt to obtain a copy of the simulation used in (2). Unfortunately, Bell Labs' policy prevented them from releasing their code. Hence, we implemented an approximation of their simulation to further explore DFFR.

Our simulations use a cost which is equal to the power used. We also impose a hard limit on the number of users assigned to each subband and the amount of power used. When considering the supportable number of users we use the lack of subcarrier availability or excess power consumption as our stopping criterion.

In the simulations involving randomness (user placement and/or fading), we executed each multiple times and averaged the results.

[1]The cost is a function of the price for power on subband j, how much power would be required (p_{ij}), the number of users already assigned to subband j by this base station, and the amount of power already used by the base station. Details can be found in (2).

4.1 Analysis: comparison to optimal configuration

A two-cell toy scenario was presented in Figure 2 and we explored the utility of different resource allocation schemes for this in Figure 3. In this section we test the power minimization heuristic on the same toy example (again without fading) but restrict ourselves to a constant data rate scenario (for voice users: 9.6 kbps). In Figure 5 we see how the resource allocation works as we vary the total power constraint on the BS and the distance of the two users from their BS. In this case we are looking the parallel of a slice of the 3D Figure 3 (since the total system capacity achieved is constant). When d is small and both users are close to their base stations the algorithm converges to universal reuse (green region). When d is large, and both users are on the cell-edge, the algorithm implements reuse-$\frac{1}{2}$ as expected (blue region). The orange region is the set of power-distance pairs where the CDR can be satisfied by either universal reuse or reuse-$\frac{1}{2}$ and the differences in power requirements are not significant. The red region indicates when the BS power constraint is hit and the CDR is not achieved. Users in this regime would be in outage (they are too far away to be served with such a small power constraint even when they have a clean channel).

We see that the behavior of the power minimization heuristic matches our intuition based on the toy example.

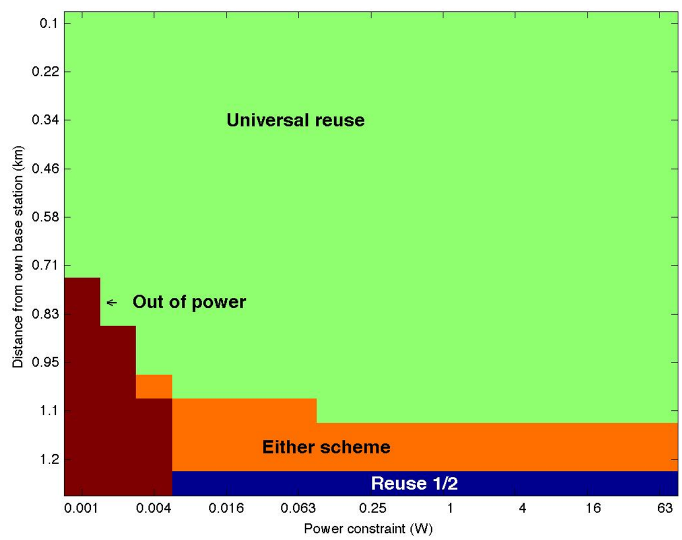


Figure 5: Performance of power minimization algorithm for the two-cell, two-subband, two-user case of Figure 2

4.2 Analysis: number of users supported

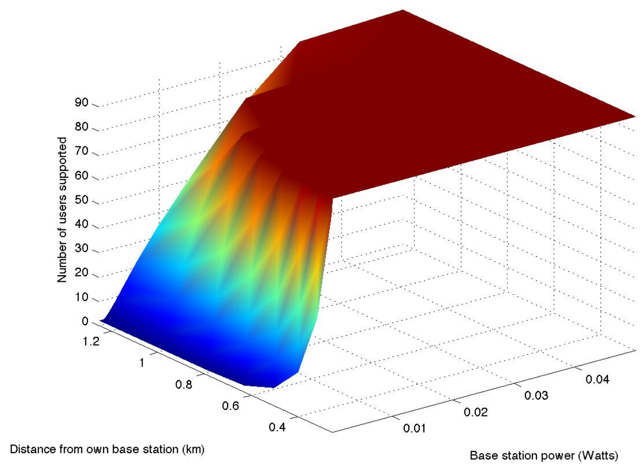
For this analysis we make the following assumptions:

- Path loss model: $133.6 + 35\log(d)$
- Two cells with inter-site distance 2.5 km
- Total bandwidth: 1.25 GHz
- 48 subcarriers; 1 subcarrier per subband
- Each person may only use one subcarrier
- Users desire a constant data rate of 9.6 kbps
- Fades are Rayleigh with parameter 1 and are independent across subcarriers
- Slow fading

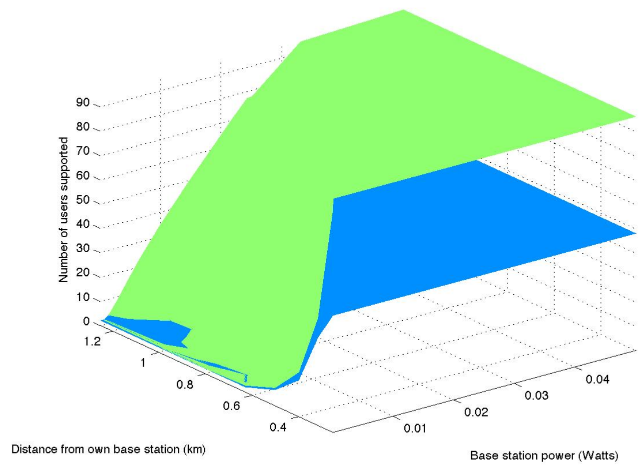
We add users one by one while letting the system stabilize in between. If the system does not stabilize in time, we assume that we cannot support the newest user and stop the simulation. The simulation also ends if we exceed the number of subcarriers or the total power available.

In Figure 6a we consider the two-cell case as in Figure 2 where new users are added at locations A and B. Figure 6a shows the empirical number of supportable users using DFFR. We see that in many cases we are limited only by the number of subcarriers in our system. Figure 6b compares DFFR to reuse-$\frac{1}{2}$ and we see that DFFR is clearly better by this measure. Further detail in the form of slices of Figure 6a can be found in Figure 7a.

Figure 7b shows the effect of user location. "Inner" refers to the situation where there are groups of users exactly 0.5 km from their respective base stations (cell interior) whereas in the case "outer" these groups are at 1.25 km (cell edge). "Inner and outer" is the natural combination of these two situations (half of the users are at 0.5 km and half are at 1.25 km). "Uniform" places users randomly between their base station and the cell edge.

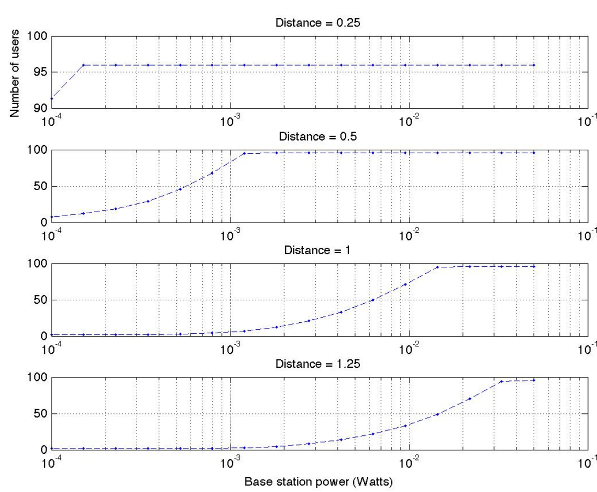


(a) Effect of location and power constraint on the supportable number of users under DFFR

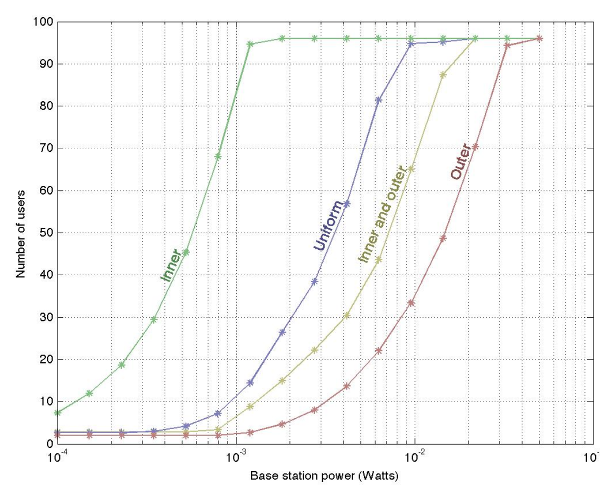


(b) Comparison of DFFR (green) and reuse-$\frac{1}{2}$ (blue)

Figure 6: Dynamic FFR with varying power and distance from BS



(a) Slices of Figure 6a



(b) Effect of user distribution within the cell and total power constraint on the number of users the system can support

Figure 7: Dynamic FFR with varying power and distance from BS

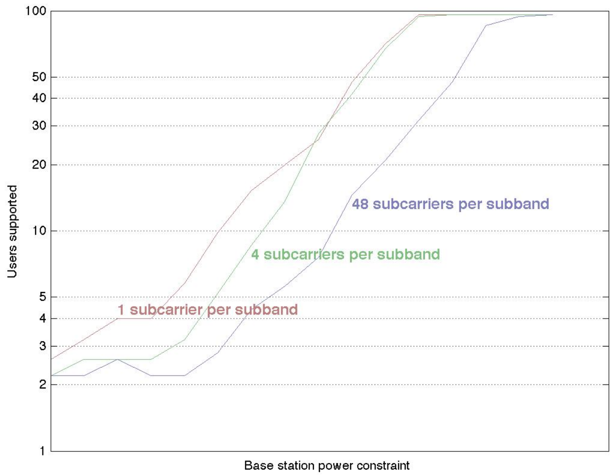


Figure 8: Effect of subbands on the number of users the system can support

Finally, Figure 8 highlights the importance of subbands on the supportable number of users. In the case where there is one subcarrier per subband, the noise is highly organized: each user gets to hand-pick his interferer. This is what we found was optimal in the example of Figure 4. In the case where there are forty-eight subcarriers per subband, the noise is disorganized and each user sees interference from all of the users in the other cell.

4.3 Analysis: power usage

Since the goal is to minimize power usage rather than maximize capacity, it is natural to examine the amount of power saved. Table 1 shows the magnitude of this effect in a case study. The system is as described above (two cells, 48 subcarriers) but with precisely 20 users per cell. In this case, the power savings is not significant on the dB scale and was representative of all of the scenarios we examined.

	Reuse-$\frac{1}{2}$	DFFR	Difference (% mW)
Maximum power	.81 mW (29 dBu)	.55 mW (27 dBu)	32%
Average power	.19 mW (22 dBu)	.15 mW (22 dBu)	21%
Total power	7.8 mW (39 dBu)	6.1 mW (38 dBu)	21%

Table 1: Power usage

4.4 Analysis: outage probability

The authors of (4) previously considered the capacity and outage probabilities under an (different) FFR scheme. Within a CDR framework, considering maximum system capacity makes no sense and we analyze the worst case outage probability for the DFFR algorithm described. Each user experiences independent Rayleigh fading on each subcarrier. Here we compare the worst-case outage probability experienced by a user using a DFFR scheme and that of a user in a reuse-$\frac{1}{2}$ scheme.

Required rate	R
Max Tx power from BS to one user	P_{max}
Thermal noise	N
Bandwidth per subcarrier	W
Max possible SNR	$SNR_{max} = \frac{P_{max}}{N}$
Number of users in each cell	n_{cell}
Number of subcarriers in system	n_s
Fade from BS-k to Bob on subcarrier j	h_{1j}

Each user $i, 0 \leq i \leq n_{cell}$ can be assigned to only one subcarrier, though this analysis could be extended (not so easily) to a situation where each user is allowed to transmit on multiple subcarriers. We consider the outage of user Bob in cell 1 and assume users in all neighboring cells have already been allocated subcarriers. Consider the case when $n_{cell} < \frac{n_s}{2}$ so that there are many subcarriers for users to choose between. Given a particular user (e.g. Bob), let $\mathbf{A}, \mathbf{B}, \mathbf{C}$ denote the following sets of subcarriers:

$\mathbf{A} = \{j \mid 0 \le j \le n_s$, subcarrier j is assigned to a user other than Bob in cell 1$\}$

$\mathbf{B} = \{j \mid 0 \le j \le n_s$, power $> P_{max}$ would be required to get rate R in subcarrier j $\}$

$\mathbf{C} = \mathbf{A} \cup \mathbf{B}$

Thus, C is the set of all subcarriers unavailable to Bob.

We assume that $j \in \mathbf{A}$ and $j \in \mathbf{B}$ are independent events. This is reasonable since all users in the system see independent fades on each of the subcarriers. Also, we assume that users are uniformly spread within a cell and thus the interference they observe can be considered independent. (This assumption is more easily justified when there are multiple interfering cells surrounding the given cell, however, our analysis only considers one neighboring cell.) Finally, we assume that $j \in \mathbf{C}$ and $k \in \mathbf{C}, 0 \le j,k \le n_s, j \ne k$ are independent events. (In a system where $n_s \sim n_{cell}$, this is clearly not true: there will be strong correlations between the members of $\mathbf{A}$ since no user in cell 1 will choose a subcarrier with high interference from neighboring cells. However, here we assume there is no such limitation.)

4.4.1 DFFR

Let us now consider $P(j \in \mathbf{A})$ and $P(j \in \mathbf{B})$. With n_s large enough, other users in cell 1 can be considered to be uniformly distributed among subcarriers. Hence,

$$P(j \in \mathbf{A}) = \frac{n_{cell} - 1}{n_s} \tag{2}$$

There are two cases to be considered to understand $\mathbf{B}$.

$$\begin{aligned}
P(j \in \mathbf{B}) &= P\left(\frac{||h_{1j}||^2}{N + \sum_k ||h_{kj}||^2 \cdot P_{max}} < \frac{2^{\frac{R}{W}} - 1}{P_{max}}\right) && (3)\\
&= P\left(\frac{||h_{1j}||^2}{N + \sum_k ||h_{kj}||^2 \cdot P_{max}} < \frac{2^{\frac{R}{W}} - 1}{P_{max}} \;\middle|\; N > \sum_k ||h_{kj}||^2 \cdot P_{max}\right) \cdot P\left(N > \sum_k ||h_{kj}||^2 \cdot P_{max}\right) && (4)\\
&\quad + P\left(\frac{||h_{1j}||^2}{N + \sum_k ||h_{kj}||^2 \cdot P_{max}} < \frac{2^{\frac{R}{W}} - 1}{P_{max}} \;\middle|\; N \le \sum_k ||h_{kj}||^2 \cdot P_{max}\right) \cdot P\left(N \le \sum_k ||h_{kj}||^2 \cdot P_{max}\right)
\end{aligned}$$

The analysis while considering interference from multiple surrounding cells becomes very complicated and we cannot get explicit expressions for the probabilities (since the pdf of the ratio of a Rayleigh and gamma random variable does not have a clean solution). Hence, we consider the case with only one interfering cell, say cell 2.

$$\begin{aligned}
P(j \in \mathbf{B}) &= P\left(\frac{||h_{1j}||^2}{N + ||h_{2j}||^2 \cdot P_{max}} < \frac{2^{\frac{R}{W}} - 1}{P_{max}} \;\middle|\; N > ||h_{2j}||^2 \cdot P_{max}\right) \cdot P\left(N > ||h_{2j}||^2 \cdot P_{max}\right) && (5)\\
&\quad + P\left(\frac{||h_{1j}||^2}{N + ||h_{2j}||^2 \cdot P_{max}} < \frac{2^{\frac{R}{W}} - 1}{P_{max}} \;\middle|\; N \le ||h_{2j}||^2 \cdot P_{max}\right) \cdot P\left(N \le ||h_{2j}||^2 \cdot P_{max}\right)\\
&\approx P\left(\frac{||h_{1j}||^2}{N} < \frac{2^{\frac{R}{W}} - 1}{P_{max}}\right) \cdot P\left(N > ||h_{2j}||^2 \cdot P_{max}\right) && (6)\\
&\quad + P\left(\frac{||h_{1j}||^2}{||h_{2j}||^2 \cdot P_{max}} < \frac{2^{\frac{R}{W}} - 1}{P_{max}}\right) \cdot P\left(N \le ||h_{2j}||^2 \cdot P_{max}\right)\\
&= \frac{2^{\frac{R}{W}} - 1}{SNR_{max}{}^2} + \frac{(2^{\frac{R}{W}} - 1)^2}{(2^{\frac{R}{W}} - 1)^2 + 1}\left(1 - \frac{1}{SNR_{max}}\right) && (7)
\end{aligned}$$

Equation 7 follows since $P(||h||^2 < \frac{1}{\alpha}) \approx \frac{1}{\alpha}$, and $P(\frac{||h_1||^2}{||h_2||^2} < q) = \frac{q^2}{q^2+1}$ (derivation omitted for brevity).

Using this, we can now calculate the outage probability for Bob as

$$P(j \in \mathbf{C}) = P(j \in \mathbf{A}) + P(j \in \mathbf{B}) - P(j \in \mathbf{A})P(j \in \mathbf{B}) \tag{8}$$

$$\text{P(User Bob in outage)} = \prod_{j=1}^{n_s} P(j \in \mathbf{C}) \tag{9}$$

We note that in the high SNR regime the outage probability is independent of the SNR and is primarily determined by $\frac{R}{W}$ and $\frac{n_{cell}}{n_s}$. To understand the nature of the probability we consider a case study with $\frac{n_{cell}}{n_s} \approx \frac{1}{4}$ and high SNR_{max}. In this case

$$\begin{aligned}\text{P(Bob in outage)} &\approx \left(\frac{1}{4} + \frac{3}{4}Pr(j \in \mathbf{B})\right)^{n_s} \\ &= \left(\frac{1}{4} + \frac{3}{4}\left(\frac{2^{\frac{R}{W}}-1}{{SNR_{max}}^2} + \frac{(2^{\frac{R}{W}}-1)^2}{(2^{\frac{R}{W}}-1)^2+1}\left(1 - \frac{1}{SNR_{max}}\right)\right)\right)^{n_s}\end{aligned}$$

4.4.2 Reuse-$\frac{1}{2}$

We similarly calculate for a two-cell setup:

$$P(j \in \mathbf{A}) = \frac{n_{cell} - 1}{\frac{n_s}{2}} \tag{10}$$

$$P(j \in \mathbf{B}) = \frac{2^{\frac{R}{W}} - 1}{SNR_{max}} \tag{11}$$

$$P(j \in \mathbf{C}) = Pr(j \in \mathbf{A}) + Pr(j \in \mathbf{B}) - Pr(j \in \mathbf{A})Pr(j \in \mathbf{B}) \tag{12}$$

$$\text{Pr(User i in outage)} = \prod_{j=1}^{\frac{n_s}{2}} Pr(j \in \mathbf{C}) \tag{13}$$

Note the difference between equations 10 and 2. In the reuse-$\frac{1}{2}$ case, a given user can only choose from half as many subcarriers. To understand equation 11 we note that in a reuse-$\frac{1}{2}$ scheme users in cell 1 experience no interference from users in cell 2 and vice versa. Subcarrier j can only end up in **B** because of a bad fade, with probability as above.

Consider again the case where $\frac{n_{cell}}{n_s} \approx \frac{1}{4}$ and high SNR_{max}. In this case, we see

$$\text{Pr(User i in outage)} \approx \left(\frac{1}{2} + \frac{1}{2}Pr(j \in \mathbf{B})\right)^{\frac{n_s}{2}} \tag{14}$$

$$\approx \left(\frac{1}{2} + \frac{1}{2}\frac{(2^{\frac{R}{W}}-1)^2}{SNR_{max}}\right)^{\frac{n_s}{2}} \tag{15}$$

4.4.3 Outage comparison: DFFR and reuse-$\frac{1}{2}$

Based on the case study we can see the the DFFR outage probability has n_s as the exponent, while the reuse-$\frac{1}{2}$ outage probability has $\frac{n_s}{2}$. Thus, the DFFR outage probability decays much faster than the reuse-$\frac{1}{2}$ probability. We can say that DFFR gains a <u>subcarrier diversity</u> over reuse-$\frac{1}{2}$: twice as many subcarriers need to be bad before the user is forced into outage.

5 Acknowledgments

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